An Efficient Technique For The Time Domain Analysis of Multi-conductor Transmission Lines Using The Hilbert Transform

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Abstract

Most models that appeared in the literature for the transient time domain analysis of lossy multi-conductor, multi-dielectric transmission line systems are non causal and fail to accurately predict the pulse distortion resulting from the losses in a multi-conductor transmission line for very fast digital signals. The reason has been found in the modeling of the frequency dependent material characteristics, particularly the complex dielectric constant $\epsilon(\omega)$ [1]. In this paper, a causal model, based on the Hilbert Transform, is presented.

1 Introduction

It has been found [1] that in order to obtain an accurate causal response for the multi-conductor transmission line, extremely accurate characterization of the complex dielectric permittivity is required. The accuracy thought is of the order of 5 % or less. Such accuracy is generally unattainable from currently known broad band measurement techniques [2-6] . In this paper, the Hilbert Transform has been used to enforce the causality requirements on the time dependent TEM fields of the transmission line. It is shown that it is possible to very accurately compute the time domain response of a multi-conductor transmission line without very accurately knowing the frequency dependent material properties.

2 Frequency Domain Modal Analysis of Multi-Conductor Transmission Lines

It is commonly known [7-15] , that on an N signal and one return multi-conductor line, there are N independent TEM modes that propagate on each line, each mode is independent from all other modes and characterized by it propagation constant γ_i and its amplitude V_{0i}. The propagation constants of the modes are determined from an eigen-value problem [7,8] given by

$$Det\left\{\gamma_i^2[I] - ([R] + j\omega[L])\,([G] + j\omega[C])\right\} = 0 \tag{1}$$

Where [L], [C], [R], and [G] are the line circuit parameters, namely the per unit length inductance, electrostatic induction, resistance and conductance matrices [16-20]. The resistance matrix [R] is associated with the conductor loss and is assumed to be zero in this paper. Once equation (1) is solved, the voltages at any point on the line can then be expressed as the sum of the N modes, each mode being expressed as

$$V_i(\omega) = V_{0i}\, e^{-j\omega\, \beta_i(\omega)\, \frac{L}{c_0}}\, e^{-\omega\, \alpha_i(\omega)\, \frac{L}{c_0}} \tag{2}$$

Where the subscript i denotes the i^{th} mode , $\alpha_i(\omega)$ and $\beta_i(\omega)$ are real functions of frequency such that

$$\gamma(\omega) = \alpha(\omega) + j\beta(\omega) \tag{3}$$

L is the line length and c_0 is the velocity of light in free space. From the theory of signal processing [21] , it is well known that the transfer function of any causal system can be viewed as a cascade of an all pass and a minimum phase system. It is also well known that the loss function of the transmission line is a minimum phase function [11] . The expression for the modal voltages on the line can then be rewritten as

$$V_i(\omega) = V_{0i} e^{-j\omega\beta_{0i}\frac{L}{c_0}} e^{-j\omega\delta_i(\omega)\frac{L}{c_0}} e^{-\omega\alpha_i(\omega)\frac{L}{c_0}} \tag{4}$$

Where $\beta_i(\omega) = \beta_{0i} + \delta_i(\omega)$. Thus the term $(e^{-j\omega\beta_{0i}\frac{L}{c_0}})$ denotes the all pass linear phase system, while the term

$(e^{-j\omega\delta_i(\omega)\frac{L}{c_0}} \, e^{\omega\alpha_i(\omega)\frac{L}{c_0}})$ denotes the minimum phase system. It is also known that if a system is minimum phase, then the log of its magnitude and its phase are Hilbert Transform pairs [21].

3 Computation of The Frequency Dependent Dielectric Constant Using The Hilbert Transform

From the above analysis we then deduce the following procedure for accurately computing the transient time domain response of the multi-conductor transmission line.

(a) Roughly obtain $\epsilon'(\omega)$ and $\epsilon''(\omega)$ from measurements or by some other means

(b) Compute the propagation constant $\gamma_i's(\omega)$ for the various modes propagating on the line, the modal voltages are then expressed as

$$V_i(\omega) = V_{0i} e^{-j\omega\beta_i(\omega)\frac{L}{c_0}} e^{-\omega\alpha_i(\omega)\frac{L}{c_0}} \tag{5}$$

(c) Obtain $\delta(\omega)$ as, $\delta(\omega) = H[\alpha(\omega)]$, H being the Hilbert Transform.

(d) Recompute the modal voltages as

$$V_i(\omega) = V_{0i} e^{-j\omega(\beta_i(\omega) + \delta_i(\omega))\frac{L}{c_0}} e^{-\omega\alpha_i(\omega)\frac{L}{c_0}} \tag{6}$$

(e) In the case of a single transmission line, an equivalent $\epsilon(\omega)$ that corresponds to the causal response may then be extracted as :

$$\epsilon(\omega) = \{[\beta(\omega) + \delta(\omega)] - j\alpha(\omega)\}^2 \tag{7}$$

It should be noted that the non linear phase term $\delta(\omega)$ is very small compared to the linear phase term β_0 for most current practical applications and has been often neglected in most time domain analyses. This however, results in large errors in the time domain response of the line when the rise time of the signals is very small due to the multi-valuedness of the phase function [1] in equation (8).

4 Numerical Examples

In order to demonstrate the usefulness of technique we solve for the impulse response and time domain pulse response of a single microstrip line. The line geometrical and circuit data are shown in figs 1. and 2. Fig. 3. shows the impulse response of the line when the dielectric constant is given by $\epsilon(\omega) = 4.0(1 - j0.01)$ and the input rise time is 50 picosecond. The dashed curve shows the non causal response computed using the methods of [7-15], while the solid curve shows the causal and more accurate response when the Hilbert Transform is used. Similarly, the dashed curve in Fig. 4. shows the time domain pulse response of the line when the techniques of [7-15] are used while the solid curve shows the causal response obtained using the Hilbert Transform. In the non causal response the output signal starts at the load end even before the input is applied to the generator end, while in the causal response, the signal arrives at the load end at 3.3350 *nanoseconds* which is approximately the time it would take the signal to reach the load end if the line was lossless ($\frac{10}{3}$ *nanoseconds*). Fig. 5. shows the frequency dependent dielectric constant $\epsilon(\omega)$ extracted from the causal response using the Hilbert Transform. The variation from the original dielectric constant given by $\epsilon(\omega) = 4.0(1 - j0.01)$ is less then 5 % at all frequencies. Such variations are generally very difficult to measure using any known broad band measurement technique.

Finally, in order to check the accuracy of this new accurate and causal model, the theoretical data has been compared with experiment for moderately lossy lines using DuPont Pyralux Circuits. The new technique was found more accurate than the commonly known techniques.

5 Conclusion

Several authors have analyzed the time domain multiconductor transmission line problem [1-21]. None so far has addressed the causality issue in this exact manner as the error, introduced by the lack of extremely accurate frequency domain characterization of the material dielectric constant, is generally small for most current practical applications. As the trend for faster digital signals, and thus higher rise times continues, the causality issue becomes extremely important and can no longer be ignored. This technique uses the Hilbert Transform in lieu of an extremely accurate characterization of the material properties to enforce the causality requirements on the electromagnetic fields. This technique can also be used to enhance the broad band measurements techniques for the complex dielectric permittivity. Work is being done along these lines and will be reported promptly.

6 References

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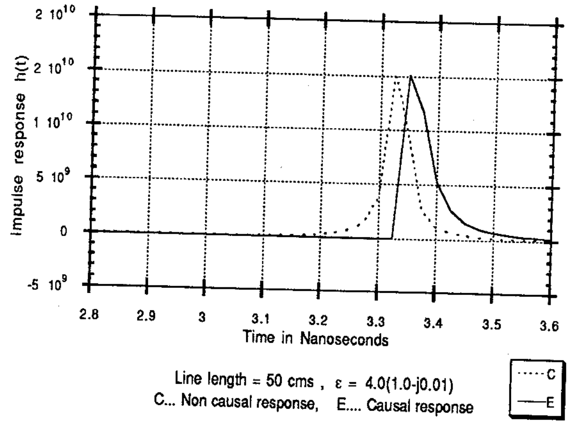
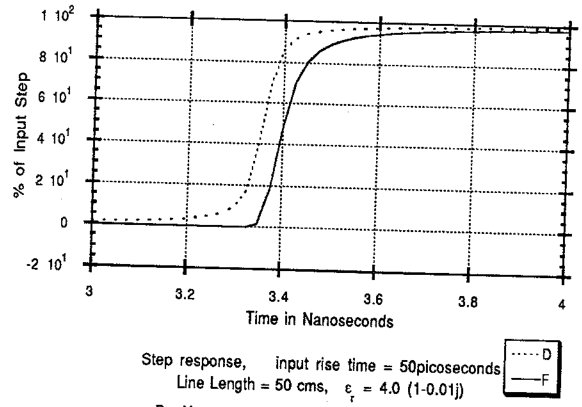


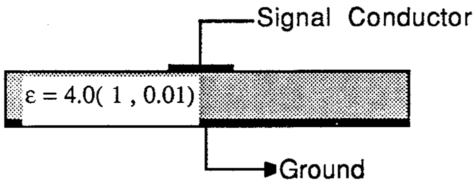
Line length = 50 cms , ε = 4.0(1.0-j0.01)
C... Non causal response, E.... Causal response

Fig. 3.



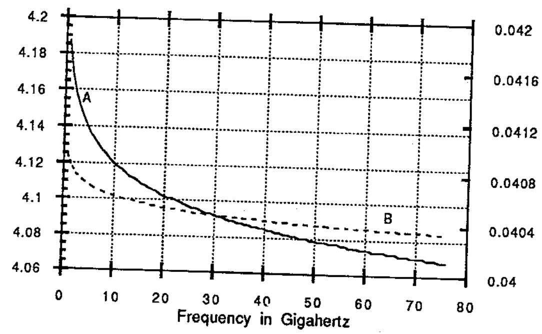
Step response, input rise time = 50picoseconds
Line Length = 50 cms, ε_r = 4.0 (1-0.01j)
D... Non causal response, F.... Causal response
Fig. 4.

Single Microstrip Transmission Line



Z0 = 50 ohms, Td = 3.333 Nanoseconds

Fig.1



Equivalent Frequency Dependent Dielectric Constant Extracted From The causal response in Fig. 4.

A...ε'_{eq} (f) , B...ε''_{eq} (f)
Fig. 5

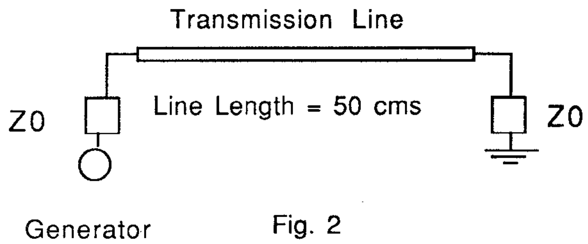


Fig. 2